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Productivity Investment and Labor Force Participation in Search Equilibrium

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Abstract

The present paper contributes to the theoretical analysis of the human capital investment and participation decision of heterogeneous workers in the search and matching framework. Its aim is to characterize the equilibrium and to identify the efficiency. Here, the paper studies search equilibrium and matching to consider the participation decision of heterogeneous workers who have different inherent ability levels. The productivity investment decision is endogenous and wages are determined by the Nash bargain among participants.

Keywords: *Education, Participation, Efficiency, Hold-up problem*

JEL Classifications: *J23, J24*

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1. Introduction

In this paper, I construct a theoretical model that focuses on the role of productivity investment incentives in the standard search and matching model. The model follows directly from insights presented by Lockwood (1986) and is an extension of the basic matching model developed by Pissarides (1990) where individuals need to invest in their productivity before participating in the labor market.

Many economists view the skills of the labor force as a prime contributor to economic performance. Therefore, not surprisingly policy makers are often interested in issues of worker training. For instance, training of less skilled workers was one of the principle policy initiatives of the first Clinton administration and the labor government in Britain has similarly made training a key policy issue.

Some OECD countries have experienced increased returns in skills investment over the last years. Many policy makers believe that low educated workers can also benefit from the changes in the demand for skills if they invest more in their education. Rosenstein-Rodan (1943), not only pointed out the importance of market demand, but also of skills, and noted that training of workers was a prerequisite for industrialization, though unlikely to happen.

While it is important to understand the optimization problem of human capital investment of an agent, it is necessary to pay attention to its subsequent interaction with labor market. Beside considering heterogeneity of workers and human capital investment and the cost related to this investment, we must also analyze how the resulting change in the supply of heterogeneous workers affects the labor market equilibrium. The interdependence becomes more complicated when we recognize there is friction in the labor market. We live in a world where information is imperfect and costly. Individual decision-making at entry (or exit) point of the market, the choice of lowest acceptable productivity and the choice of search intensity margins may be socially inefficient. What is common in all these margins, as Pissarides (1983) makes clear, is that they are all due external factors that work through the matching technology. When individuals choose to enter a market, accept a match or search more intensively they ignore the impact of their decisions on the matching probabilities of other agents in the market. Lockwood (1986) assumes heterogeneous workers in their inherent skills to show the other external effect that work through match acceptance probabilities rather than matching probabilities. In particular, he shows that the presence of high-skill workers lowers the equilibrium acceptance probabilities of low skill workers below what is socially desirable.

An aggregate increase in return to education is a standard property of matching models . With human capital investment , a more productive workforce raises the labor demand. However a tighter labor market increases the incentives to schooling . This mechanism can give rise to multiple equilibria [see e.g. Laing et al.,1995; Burdett and Smith (2002)]. They argue that if workers' decision to invest in education and firms' job creation decisions interact positively, then the economy can get stuck in a bad equilibrium. As a result the human capital investment will be insufficient and also unemployment will be high. On the other hand Acemoglu (1996), argue that frictional unemployment can create a hold up problem whereby workers are generally paid a share less than their marginal productivity . Charlot et al.(2005) show that equilibrium is unique in their framework since the productivity effect of schooling dominates the wage effect . Clearly the presence of matching unemployment is a common explanation of weakness of incentives for education investment.

Since Becker (1964) , provided the labor market is perfectly competitive, the market outcome is socially optimal. However, in the presence of unemployment , models analyzing matching models with ex post Nash bargaining and ex ante costly human capital investment Laing et al. (1995) and Acemoglu (1996), workers under-invest in education which is the result of hold up phenomenon . Clearly search frictions play an important role in that. Though Cole et al.(2001) show that there can also be over investment in the frictionless environment with two sided investment.

My argument is based on workers' heterogeneity and self-selection in education. The introduction of heterogeneity makes the model more realistic and in addition allows one to gain insights into the worker behavior that do not follow from models of homogeneous labor market . When workers differ according to the ability level and the cost of productivity level that he or she is going to attain , the worker must decide to invest in his/her productivity and then participate in the labor market . This is an integral part of participation decision.

It is well established in the literature that a core topic in labor economics is `self-selection'. The starting point of this topic in economics is Roy's (1951)

"Thoughts on the Distribution of Earnings". Self-selection means in theory, that rational individuals make optimizing decisions about what markets to participate in- job, education, crime, etc. In the present paper heterogeneity of the labor force in their abilities and education cost give rise to self-selection. Where the cost of productivity investment is not constant and it depends on ability of the worker and also the amount of investment on productivity.

Alternative specifications of the matching process have been considered in the literature: search may be undirected (see, e.g. Acemoglu, 1999; Albercht and Vroman, 2002) or skilled workers may poach on unskilled jobs (see, e.g. Gautier, 2002). Charlot and Decreuse (2005), consider two separate matching sectors where educated workers direct their search towards high productivity occupations. Adopting those different specifications would, of course, alter our results.

My argument also is related to intra/infra marginal decision. And individual decisions are characterized into two classes: intramarginal decision of resource allocation and inframarginal decision of economic organization. Intramarginal decisions involve the extent to which resources are allocated and inframarginal decisions are about what activities to engage. Considering the two types of decisions, the paper analyses a theoretical model that focuses on equilibrium incentives for productivity investment in matching framework with one-sided heterogeneity. There are two risk neutral groups: workers and employers. There are two types of workers; low ability and high ability workers while employers are identical. Distribution of abilities is exogenous.

This paper argues human capital investment decision[1] of heterogeneous workers in equilibrium search framework. The technology that I assume is such that workers invest on their education to achieve productivity and then search for a job. Of course education is costly, those who invest in education and search for job are participant in the labor market. In this paper, the choice of investment in human capital is intramarginal decision since it involves deciding the quantity of resources devoted to acquire human capital. Once he/she has chosen his investment decision in human capital, he then searches for the job.

The standard search model features exogenous labor supply-i.e., a fixed size of the labor force. Rather than consider flows between search

1. It is an investment in human capital that pays off in terms of higher productivity.

unemployment and employment of a fixed labor force , I examine the participation decisions of worker . If labor force participation is fully endogenous, just as is labor demand in the form of vacancy creation, then another condition which I name it labor force participation constraint impinges the basic framework equations . Therefore, workers reach different participation decisions on the basis of comparison between labor market and non-market returns. Here I consider a labor force participation decision, a margin which is absent in most of models of labor market. This paper exploits the basic insights of above models; i.e., self-selection at the individual level alters the equilibrium composition of groups. My contribution to this literature is then to highlight the equilibrium characteristics of heterogeneous workers with endogenous productivity investment and free entry of firms which drives job creation.

The paper uses a matching process in the spirit of Pissarides (1990) with a Nash bargaining approach to wage-setting. Given overall labor market condition , equilibrium is determined by free entry condition , optimal productivity investment decision, the participation constraint plus the steady state conditions. Equilibrium can take one of the following forms. The first type of equilibrium is one in which it is beneficial for both types of workers to be active in the labor market. In the second type of equilibrium, there is only the willingness of high ability workers to invest in their productivities and search for job. Third, for small changes in the economic environment multiple equilibria can exist. Finally I show that heterogeneity is not the cause of multiplicity in this model.

The paperturns in the next section to the presentation of the model. This is followed, in Section 3, by an analysis of the decentralized and multiple equilibrium. In section 4, I discuss the social planner problem then I focus on the three different policies that can be implemented by the planner that may achieve an efficient allocation. Finally, in section 5, I summarize my results and conclude.

2. The Model

2.1. The basic framework

The model is an extension of the Lockwood (1986) , Pissarides (2000) matching framework . The economy is composed of two risk-neutral groups : workers and firms . All firms are identical whose number is endogenously determined by a standard free entry condition but workers differ in their abilities. This heterogeneity in ability implies different workers invest in different productivity levels. Time is continuous and throughout I only consider steady state . The economy is composed of workers and employers . All employers are identical whose number is endogenously determined by a standard free entry condition. There is turnover of workers where ϕ is the inflow of new entrants and all workers die according to a Poisson process with parameter ϕ. Thus steady state implies there is a unit measure of workers in the economy.

There are two types of entrants: those with low-ability (a_1) and those with high-ability (a_h). Let $i \in l, h$ denote a worker's type where $a_h > a_l > 0$. Assuming fraction η_i of entrants are type i, the distribution of ability across entrants, denoted $G(a)$, is

$$\begin{cases} G(a) = 0 & \text{for } a < a_l \\ G(a) = \eta_1 & \text{for } a_l \leq a < a_h \\ G(a) = 1 & \text{for } a \geq a_h \end{cases}$$

Given their ability a_i, each entrant type i first invests in education which determines his/her productivity level x . Let $C(x; a^i)$ denote the cost of investing to productivity x given initial ability a_i . Assume $C(.)$ is strictly increasing, convex and twice differentiable in x. Also $C(a_i; a_i) = 0$, $C(.)$ is decreasing in a (it is less costly for a higher ability to achieve a given productivity level) and $C_{xa} < 0$ so that higher ability types face a lower marginal cost to achieving a higher productivity level x. For ease of exposition assume the Inada condition $C_x(a_i, a_i) = 0$.

The model has a standard hold-up structure: given beliefs on market wages, an individual who is born with ability a first selects a productivity level x. The cost of obtaining that productivity level depends on ability a. After investing in her productivity, she then enter the labor market and search for a job. If she does so and contacts a firm, her wage is then determined by Nash bargaining. Of course expectations are rational: the negotiated wage is consistent with her original beliefs. As firms are identical, $w = w^*(x)$ will denote the equilibrium negotiated wage.

After investing in productivity x, a type i entrant decides whether to search or not with effort $e \in \{0,1\}$. I define those who choose search $e = 1$ as active job seekers , all others are inactive (non-participant) . Clearly as education is costly, those who choose to be inactive will also choose zero education. Conversely the Inada condition ensures those who are active choose a strictly positive education level. In steady state, let $1-\pi$ denote the proportion of active agents who are high ability (π will be endogenously determined and depending on the investment choice of workers).

E_i denotes the number of employed workers with ability a_i. To fill a job, an employer must first create a vacancy at flow cost k. If V denotes the number of vacancies and $U = U_l^A + U_h^A$ the number of active unemployed job seekers , then the match flow is described by a matching function $M = M(U,V)$ which is increasing in both arguments and has constant returns. Let $\theta = \frac{V}{U}$ denote market tightness. As active job seekers meet vacancies at rate $\frac{M}{U}$, standard arguments imply this job contact rate is $m(\theta) \equiv M(1,\theta)$ and $m(.)$ is an increasing concave function. Similarly $\frac{m(\theta)}{\theta}$ is the rate at which a firm holding a vacancy contacts an active job seeker. Random search implies $(1-\pi)\frac{m(\theta)}{\theta}$ is the rate at which a firm contacts an active job seeker with high ability.

When a firm holding a vacancy and an active job seeker meet, the worker's productivity x is observed and they bargain over the wage. Wages are determined by Nash bargaining. Job matches break up at an exogenous rate δ in which case the worker returns to the pool of unemployed workers and, with free entry, the firm makes no further profit.

2.2. Worker's payoffs and job search strategies

Before describing optimal productivity choice, I first describe the expected lifetime value of being unemployed with productivity x in a market with tightness θ, which I denote $V_U(x,\theta)$. Below Nash bargaining will yield a negotiated wage outcome which I denote $w = w^N(x,\theta)$. Standard turnover arguments imply the value of being an active worker with productivity x satisfies

$$(r+\phi)V_U(x,\theta) = b + m(\theta)[V_E(x,\theta) - V_U(x,\theta)] \tag{2.1}$$

where $V_E(x,\theta)$ describes the value of being employed with productivity x and tightness θ is given by

$$(r + \phi)V_E(x, \theta) = w^N(x, \theta) + \delta[V_U(x, \theta) - V_E(x, \theta)], \quad (2.2)$$

an active job seeker enjoys flow payoff b and finds employment at rate $m(\theta)$ with associated gain $V_E(x, \theta) - V_U(x, \theta)$. While employed, the worker negotiates wage $w = w^N(x, \theta)$ as determined below. At rate δ the job is exogenously destroyed and the worker returns to the pool of unemployed workers. Substituting out $V_E(.)$, these equations imply:

$$V_U(\mathrm{x}, \theta) = \frac{(\mathrm{r} + \phi + \delta)\mathrm{b} + \mathrm{m}(\theta)w^N(\mathrm{x}, \theta)}{(\mathrm{r} + \phi)(\mathrm{r} + \phi \ + \mathrm{m}(\theta) + \delta)} \quad (2.3)$$

Clearly, the worker with productivity x will only enter the labor market if and only if $V_E(x, \theta) \geq V_U(x, \theta)$. As education is costly, those who choose not to enter the labor market, the non-participants, will choose $x = 0$.

2.3. Productivity investment decision

In this section the equilibrium market outcome is taken as given. Specifically as each worker is small, he/she takes the market tightness parameter θ as given. Also he/she anticipates the equilibrium wage that is negotiated by a worker of productivity x Below this is denoted $w = w^N(x, \theta)$. Thus given innate ability a_i, market tightness θ and wage function $w^N(x, \theta)$, the worker first chooses the optimal level of productivity x to maximise the expected value of lifetime utility. To solve this problem, I let $V_U(x, \theta)$ denote the expected discounted value of lifetime utility for an unemployed active worker with productivity x. Optimal productivity of type i worker, denoted $x_i^* \equiv x^*(a_i, \theta\ )$ conditional on being active is then given by;

$$x^*(a_i, \theta\ ) = \arg\max_{x \geq a_i}[V_U(x, \theta\ ) - C(x; a_i)] \quad (2.4)$$

The RHS is the sum of value of being active unemployed minus the direct cost of investment in the productivity. The Inada condition ensures the necessary condition for optimal x_i^* is given by;

$$\frac{\partial V_U(x_i^*, \theta)}{\partial x} = \frac{\partial C(x_i^*; a_i)}{\partial x} \quad (2.5)$$

Of course this condition describes the optimal investment choice for active workers - those who will choose to enter the labor market and search for employment. Not all workers, however, will choose to be active. In equilibrium, there is a critical ability a^c where those with ability $a_i < a^c$ will choose not to be active. Specifically by staying out of the labor market, each worker can always generate payoff $b/(r+\phi)$. Thus only workers whose participation in the labor market exceeds $b/(r+\phi)$ will be active labour market members.

Definition of $a^c(\theta)$:

$$V_U(x^*,\theta) - C(x^*,a^c) = \frac{b}{r+\phi} \quad (Active; Constraint) \tag{2.6}$$

where $x^* = x^*(a^c,\theta)$ is the optimal productivity choice of an active participant with ability $a = a^c$. Note that a^c depends on market tightness θ - below we shall show that a^c is decreasing in θ; i.e., higher market tightness leads to more (low ability) workers choosing to become active. Claim 1 now establishes that a worker of type i is active in the labour market if and only if ability $a_i \geq a^c(\theta)$.

***Claim 1*:** For any θ:

(i) Individuals with $a_i \geq a^c(\theta)$ *are active, and choose* $x = x^*(a_i,\theta)$

(ii) Individuals with $a_i < a^c(\theta)$ *are inactive, choose* $x = a_i$ *at zero cost and enjoy* $\frac{b}{r+\phi}$.

Proof:

As an active worker of ability a solves the program

$$\max_{x \geq a_i}[V_U(x,\theta) - C(x;a)],$$

the Envelope theorem implies this payoff is strictly increasing in a As worker $a = a^c$ is indifferent to participating, then all those with $a_i > a^c$ strictly prefer to participate (and invest to $x^*(a_i,\theta)$ while all those with $a_i < a^c(\theta)$ strictly prefer not to participate (and so choose $x = a_i$). Having described the optimal investment decision of workers, the next step is to compute the value of being unemployed in a market equilibrium with tightness θ.

2.4. Steady state turnover

In order to solve the free entry condition for equilibrium market tightness, we first describe steady state turnover. Suppose type i have ability $a_i > a^c(\theta)$ and so are active labour market participants. Recall that E_i was defined as the number of type i workers who are employed and U_i the number who are (active) unemployed. Steady state implies

$$(m(\theta) + \phi)U_i = \delta E_i + \phi\eta_i \tag{2.7}$$

where the LHS describes the flow of i type workers out of unemployment, while the inflow is composed of employed workers who lose their jobs and those new market entrants who are type i As $E_i + U_i = \eta_i$ this implies the number of active type i unemployed worker is

Similarly for the pool of employed workers : Note then that if both $a_l, a_h > a^c(\theta)$, then the fraction of (active) unemployed workers who are type i is:

$$\frac{U_i}{U_l + U_h} = \eta_i$$

As π denotes the fraction of active unemployed workers who are low ability, then $\pi = \eta_l$ in this case. Conversely $a_h > a^c(\theta) > a_l$ implies low ability types are not active in the labour market. As $U_l = 0$, this implies $\pi = 0$: all active unemployed workers are high ability. This sorting effect plays an important part in what follows.

2.5. Wage determination

The paper now determines the equilibrium wage function $w^N(x, \theta)$. Again consider type i with ability $a_i > a^c(\theta)$ and so are active labor market participants. Suppose such a worker invests to productivity x and so enjoys value $V_U(x, \theta)$. Let $J_F(x, \theta)$ denote the firm's value of employing a worker with productivity x and $J_V(\theta)$ denote the value of a vacancy. Given $\beta \in (0,1)$ describes the worker's bargaining power, Nash bargaining implies the negotiated wage satisfies

$$\beta[J_F(x, \theta) - J_V(\theta)] = (1 - \beta)[V_E(x, \theta) - V_U(x, \theta)].$$

Of course a free entry equilibrium implies $J_V(\theta) = 0$ while

$$J_F(x,\theta) = \frac{x - w^N(x,\theta)}{r + \delta + \phi} \tag{2.9}$$

as the job is closed only in the event that the worker dies or the job is destroyed . By also substituting out $V_E(x,\theta) - V_U(x,\theta)$ using the above, equilibrium Nash wage agreement is:

$$w^N(x,\theta) = \frac{\beta(r + \phi + m(\theta) + \delta)x + (1-\beta)(r + \phi + \delta)b}{(\beta m(\theta) + r + \phi + \delta)} \tag{2.10}$$

The wage is thus a weighted of the worker's productivity x and the worker's flow value of unemployment b , where the weight depend on θ and the worker's bargaining power β. A rise in the productivity of the worker makes the size of the surplus to be shared between a firm and a worker with productivity x, bigger which causes the rise in the Nash bargaining wage.

2.6. The value of a vacancy

Market tightness determines which types of workers are active in the labor market as $a^c = a^c(\theta)$. Recall that π denotes the fraction of active unemployed workers who are type $i = l$ and that for types $a_i > a^c(\theta)$, their optimal productivity choice is $x^*(a_i, \theta)$. In a free entry equilibrium with random search, the expected value of a vacancy is:

$$rJ_V(\theta) = -k + \frac{m(\theta)}{\theta}[\pi J_F(x^*(a_l,\theta),\theta) + (1 - \pi)(J_F(x^*(a_h,\theta),\theta)] \tag{2.11}$$

where k is the flow cost of the vacancy. As $J_V = 0$ I obtain the free entry condition:

$$k = \frac{m(\theta)}{\theta}[\pi J_F(x^*(a_l,\theta),\theta) + (1 - \pi)(J_F(x^*(a_h,\theta),\theta)] \tag{2.12}$$

Equation (12) denote the free entry equilibrium condition which is one key equation of the equilibrium model.

2.7. The reduced form free entry condition

The following two claims help us to identify a solution to the free entry condition (2.12).

Proposition 1: $a^c(\theta)$ is continuous and strictly decreasing in θ

Proof. The above has established that for any active worker:

$$V_U(x,\theta) = \frac{(r+\phi\ +\delta)b + m(\theta)w^N(x,\theta)}{(r+\phi)(r+\phi+m(\theta)+\delta)}$$

with

$$w^N(x,\theta) = \frac{\beta(r+\phi\ +m(\theta)+\delta)x + (1-\beta)(r+\phi+\delta)b}{\beta m(\theta)+r+\phi+\delta}$$

Clearly V_U is linearly increasing in x. Some algebra also establishes that V_U is increasing and continuously differentiable with θ Now a^c is defined by

$$V_U(x^*,\theta) - C(x^*,a^c) = \frac{b}{r+\phi}\ \ (Active\ Constraint) \qquad (2.13)$$

with $x^* = x^*(a^c,\theta)$ given by

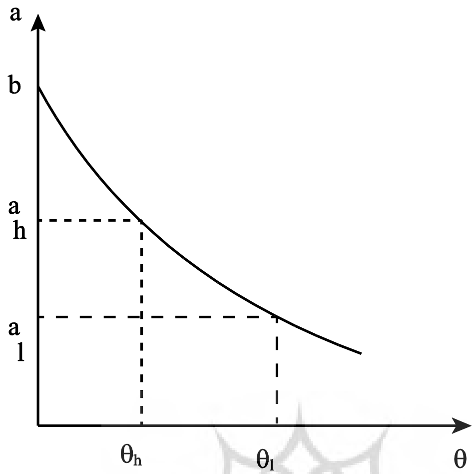
$$\frac{\partial V_U(x^*,\theta)}{\partial x} = \frac{\partial C(x^*;a^c)}{\partial x}. \qquad (2.14)$$

Totally differentiating equation (13) w.r.t θ and using (14) implies:

$$\frac{da^c}{d\theta} = \frac{\frac{\partial V_U}{\partial\theta}}{\frac{\partial C}{\partial a^c}} < 0$$

as required.

Figure 1: Critical Ability and Market Tightness



Claim 2: The Nash wage $w^N(x,\theta)$ is continuous and strictly increasing in θ

<u>Proof</u>: Trivial by differentiating the above solution for $w^N(.)$ w.r.t θ We now identify a solution to the free entry condition. Substituting out $w^N(.)$ in the equation for $V_U(.)$ yields

$$V_U(x,\theta) = \frac{b(r+\phi+\delta)+m(\theta)\beta x}{\big(r+\phi+\delta+\beta m(\theta)\big)(r+\phi)} \tag{2.15}$$

As V_U is linear in x while $C(.)$ is convex, then for $a_i \geq a^c$, the first order condition for $x^*(a_i,\theta)$:

$$\frac{m(\theta)\beta}{\big(r+\phi+\delta+\beta m(\theta)\big)(r+\phi)} = \frac{\partial C(x^*;a_i)}{\partial x} \tag{2.16}$$

describes a global maximum. Note further for $a_i > a^c$ this equation implies $x^*(a_i,\theta)$ is a continuous and strictly increasing function of a_i and θ. Also inserting the expression for $w = w^N(x,\theta)$ into equation (4.8) gives:

$$J_F(x,\theta) = \frac{(r+\phi+\delta)(x-b)}{(r+\phi+\delta+\beta m(\theta))} \tag{2.17}$$

Thus the free entry equation which specifies market tightness is defined by inserting the wage equation into the J_F and then substitute it out and rearranging the terms. Thus identifying a market equilibrium reduces to finding a θ which solves the equation,

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}\Big[\pi(\theta)x^*(a_l,\theta) + \big(1-\pi(\theta)\big)x^*(a_h,\theta) - b\Big] \; (The\ free\ entry\ condition) \tag{2.18}$$

where

$$\pi(\theta) = \begin{matrix} 0 & if\ a_l < a^c(\theta) < a_h \\ \eta_l & if \quad a^c(\theta) < a_l. \end{matrix}$$

Note that if $a^c(\theta) > a_h$ then there are no active labour market participants and there is no trade.

Claim 3: $x^*(a,\theta)$ is a continuous and increasing function of θ and is strictly increasing in a

<u>Proof.</u> For types $a > a^c$ who are active, their optimal investment choice $x^*(.)$ is given by the first order condition:

$$\frac{\partial V_U(x^*,\theta)}{\partial x} = \frac{\partial C(x^*;a)}{\partial x}.$$

Equation(2.16) implies

$$\frac{\partial V_U(x,\theta)}{\partial x} = \frac{\beta m(\theta)}{\big(r+\phi+\delta+\beta m(\theta)\big)(r+\phi)} \tag{2.19}$$

and so $\frac{\partial V_U}{\partial x}$ is a continuous, increasing function of θ. As $C(.)$ is twice differentiable and strictly convex in x, the implicit function theorem implies x^* is a continuous increasing function of θ Also as $C_{xa} < 0$ by assumption, then x^* must strictly increase in ability.

Claim 3 establishes that investment by active workers increases as market tightness increases, and does so continuously. Furthermore, comparing workers who are active, higher ability types invest to a strictly higherproductivity level.

3. Decentralized Equilibrium

Definition: *A market equilibrium is defined as follows:*

ME1: worker participate in the labor market if and only if $a_i \geq a^c$ *where:*

$$V_U(x^*(a^c,\theta),\theta) - C(x^*(a^c,\theta),a^c) = \frac{b}{r+\phi} \tag{3.1}$$

ME2: active participants choose optimal productivity choice x^* *where:*

$$\frac{m(\theta)\beta}{\big(r+\phi+\delta+\beta m(\theta)\big)(r+\phi)} = \frac{\partial C}{\partial x}(x^*(a_i,\theta),a_i) \tag{3.2}$$

ME3: free entry condition:

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}\big[\pi(\theta)x^*(a_l,\theta) + \big(1-\pi(\theta)\big)x^*(a_h,\theta) - b\big] \tag{3.3}$$

ME4: the proportion of active workers who are type i consistent with steady state turnover; when low-type are active then $\pi(\theta) = \eta_l$ *and when low-type is inactive* $\pi(\theta) = 0$

3.1. Existence and characterization

There are three types of possible equilibria. I define

$$\Omega(\theta) = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}\big[\pi(\theta)x^*(a_l,\theta) + \big(1-\pi(\theta)\big)x^*(a_h,\theta) - b\big] \tag{3.4}$$

which describes the expected return to creating a vacancy. Identifying an equilibrium requires finding a θ which solves $\Omega(\theta) = k$. Note that $m(\theta)$ is a continuous function of θ by assumption. The next step is to show that for $a_i > a^c$, that $x^*(a_i,\theta)$ is a continuous and increasing function of θ

Claim 4: $x^*(a_i,\theta)$ is a continuous and increasing function of θ and is strictly increasing in a_i.

Claim 4 establishes that investment by active workers increases as market tightness increases, and does so continuously. Furthermore, comparing workers who are active , higher ability types invest to a strictly higher productivity level.

Lemma 1: As $\theta \to 0$, $x^*(a_i, \theta) \to a_i$, for all $a_i > a^c$, $a^c(\theta) \to b$.

<u>Proof.</u> As $\theta \to 0$, equation (2.19) implies $\frac{\partial V_U(x,\theta)}{\partial x} = 0$. Hence as $\theta \to 0$, $x^*(a_i, \theta) \to a_i$, thus by (2.10), $V_U(x^*, \theta) \to \frac{b}{r+\phi}$ and (10) implies $a^c(\theta) \to b$.

Proposition 2: $\exists\ f \quad s.t\ \theta_i = f(a_i)$ and is strictly decreasing with θ_i with $\theta_i = 0$ at $a_i = b$.

<u>Proof.</u> Since $a^c(\theta_i)$ is a function of single variable and from Proposition 1 it is strictly decreasing in θ, the Inverse Function Theorem implies there exists $f(a_i) = [a^c]^{-1}(a_i)$ and

$$\left[f'(a_i) = \frac{1}{(a^c(\theta))'} = \frac{d\theta}{da} < 0 \right]$$

from Lemma 1 the proof is completed.

It is now straightforward, using Lemma 1 and proposition 2, to identify Market Equilibrium. Lets define θ_l and θ_h where

$$a_l = a^c(\theta_l)$$
$$a_h = a^c(\theta_h).$$

Note that at market tightness $\theta = \theta_i$, workers with ability a_i are indifferent between being active in the labor market and not participating. As Proposition 1 establishes that $a^c(\theta)$ is a strictly decreasing function of θ, then these definitions imply $\theta_h < \theta_l$. This then implies three possible scenarios as depicted in Figure 2:

(i) If $\theta < \theta_h$ then no workers are active in the labour market. With no loss of generality we suppose $\Omega(.) = 0$ in this region; i.e.; the expected return to creating a vacancy is zero.

(ii) If $\theta \in (\theta_h, \theta_l)$ then types $i = h$ are active as $a_l < a^c(\theta) < a_h$. As this implies $\pi(\theta) = 0$, the expected return to a vacancy is:

$$\Omega(\theta) = \frac{m(\theta)}{\theta} \frac{1-\beta}{r+\phi+\delta\ + \beta m(\theta)} [x^*(a_h, \theta) - b].$$

Note that implies $\Omega(.)$ is continuous in this range . Its slope is ambiguous, however, as $x^*(a_h,.)$ is an increasing function.

(iii) if $\theta > \theta_l$, then all types are active as $a^c(\theta) < a_l$. As this implies $\pi(\theta) = \eta_l$, the expected return to a vacancy is:

$$\Omega(\theta) = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}[\eta_l x^*(a_l,\theta) + \eta_h x^*(a_h,\theta) - b].$$

Of course $\Omega(\theta)$is not continuous in θ at θ_l,θ_h. Clearly $\Omega(.)$ increases by a discrete amount at θ_h as $\Omega = 0$ for $\theta < \theta_h$. At θ_l, however, it is easy to see that $\Omega(.)$ decreases by a discrete amount. The discontinuity is caused by low types switching to being active and, by Claim 4, their productivity $x^*(a_l,.) < x^*(a_h,.)$. To be more precise lets have a look at the each regions in details. A critical step is to note that the nature of equilibrium depends on the continuity of the right hand side of (22), i.e. $\Omega(\theta, a_i)$. Clearly as $\pi(\theta)$ is not continuous at $a^c = a_i$ for $i = l, h$ then $\Omega(.)$ is not continuous at that points.

Claim 5: $\exists\ \theta_l\ s.t.\ l\ types\ drop\ out\ if\ \theta < \theta_l$.

Claim 6: $\exists\ \theta_h\ s.t.\ h\ types\ drop\ out\ if\ \theta < \theta_h$.

Claim 7: *Given the above claims there exist three types of equilibrium:*

<u>*Region 1:*</u> The first is an equilibrium in which it is not beneficial for low/high ability worker to invest in their productivity since $\theta \le \theta_h$ i.e. $a_l, a_h < a_c$ which implies $\theta = 0$ is equilibrium and clearly everyone are inactive in this case. This is “autarchic equilibria” where workers do not participate to the labor market and as a result, firms do not post vacancies. The main concern of this paper is to focus on “non-autarchic” equilibria.

<u>*Region 2:*</u> The second is the region that $\theta_h \le \theta < \theta_l$, which shows that only high-ability type are active as $a_l < a^c < a_h$ and accordingly, $\pi(\theta) = 0$; the proportion of active low ability workers is zero. Lets show the equilibrium equations for this case are as follows:

$$\frac{m(\theta)\beta}{\big(r+\phi+\delta+\beta m(\theta)\big)(r+\phi)} = \frac{\partial C(x^*(a_h,\theta);a_h)}{\partial x} \tag{3.5}$$

$$V_U(x^*(a_l,\theta),\theta) - C(x^*(a_l,\theta);a_l) < \frac{b}{r+\phi} \tag{3.6}$$

$$V_U(x^*(a_h,\theta),\theta) - C(x^*(a_h,\theta);a_h) > \frac{b}{r+\phi} \tag{3.7}$$

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}(x^*(a_h,\theta)-b) \tag{3.8}$$

from the participation constraint for the above region it is clear that it is not beneficial for low ability type to invest to her/his productivity. One can show that the LHS of (26) is increasing in market tightness. The low ability type takes life as leisure and accordingly they will not participate in the labor market.

Region 3: The third is where $\theta_l \leq \theta$ which implies $a_l, a_h > a^c$. This region illustrates a labor market that both types are active and participate in the labor market as critical ability is lower even from the ability of low ability worker. I term such a steady state equilibrium a "Joint Type Equilibrium". This requires:

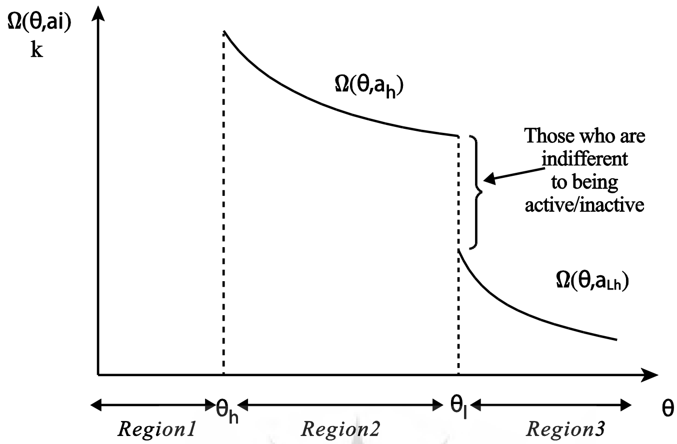
$$\frac{m(\theta)\beta}{\left(r+\phi+\delta+\beta m(\theta)\right)(r+\phi)} = \frac{\partial C(x^*(a_l,\theta);a_l)}{\partial x} \tag{3.9}$$

$$V_U(x^*(a_l,\theta) - C(x^*(a_l,\theta);a_l) > \frac{b}{r+\phi} \tag{3.10}$$

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}[\eta_l x^*(a_l,\theta) + (1-\eta_h)x^*(a_h,\theta) - b] \tag{3.11}$$

$\Omega(.)$ and k are illustrated on the vertical axis and market tightness is shown on the horizontal axis in Figure 2. The place that the critical ability meets the ability of high type corresponds to the market tightness in the region 2 and accordingly, $\Omega(.) = \Omega(\theta, a_h)$. The place that the critical ability meets the ability of low type corresponds to market tightness in the region 3 with $\Omega(.) = \Omega(\theta, a_{lh})$. Of course if the critical ability is really high even higher than the ability of high type then no one participate and region 1 represent that on the Figure 2.

Figure 2: The Equilibrium Value of Market Tightness



Of course, as set above, at the point where $\theta = \theta_l$ subject to $a^c = a_l$, the gap between the two graphs illustrates those workers that are indifferent being active or inactive.

In order that this type of Equilibrium occurs, it must be worthwhile for just high-ability active unemployed to participate in the labor market; $V_U(x^*(a_l,\theta),\theta) - C(x^*(a_l,\theta);a_l) < \frac{b}{r+\phi}$ must hold. Similarly, Joint Type equilibrium requires $V_U(x^*(a_l,\theta) - C(x^*(a_l,\theta);a_l) > \frac{b}{r+\phi}$. Mixed Strategy arises because at $\theta = \theta_l$ there exist some workers with $a^c = a_l$ who are indifferent being active or inactive.

3.2. Multiple equilibria

Multiple equilibria, can arise if the Ω function is increasing at θ. Lets look at the active constraint equation at $\theta = \theta_h$

$$\frac{b}{r+\phi} < \frac{(r+\phi+\delta)b + m(\theta)w(x)}{(r+\phi)(r+\phi+m(\theta)+\delta)} - C$$

Substituting $w(x)$ and rearranging, we have

$$C < \frac{m(\theta)\beta(x^*(a,\theta) - b)}{(r+\phi)(\beta m(\theta) + r + \delta + \phi)}$$

where $C = \frac{m(\theta_h)\beta(x^* - b)}{(r+\phi)(\beta m(\theta_h)+r+\delta+\phi)}$, inserting C into the above equation and also finding x^* from the optimal productivity constraint and substituting it in the above equation we have,

$$\frac{m(\theta_h) - \theta_h m'^{(\theta_h)}}{\theta_h m'(\theta_h)} \frac{C_{xx}C}{(C_x)^2} \frac{\beta m(\theta_h) + r + \delta + \phi}{r + \delta + \phi} \leq 1 \qquad (3.12)$$

$$\xi_{m(\theta),\theta}\, \xi_{MC,C}\, \frac{\beta m(\theta_h) + r + \delta + \phi}{C_x(r + \delta + \phi)} \leq 1 \qquad (3.13)$$

where $\xi_{m(\theta),\theta}$ is the elasticity of matching with respect to the stock of vacancies and $\xi_{MC,C}$ is the elasticity of marginal cost function with respect to the cost function. Clearly to construct an example lets assume $C(x_i; a_i) = x_i^{\gamma} a_i^{-1}$, then

$$\frac{1-\alpha}{\alpha} \frac{\gamma - 1}{\gamma} \frac{\beta m(\theta_h) + r + \delta + \phi}{r + \delta + \phi} \leq 1 \qquad (3.14)$$

multiple equilibria i.e. having $\frac{\partial \Omega(\theta)}{\partial \theta} > 0$, requires the following: If the elasticity of arrival rate of vacancy to the worker i.e. α is close to one; the marginal cost of investing in productivity is more elastic with respect to productivity investment i.e. γ and also worker's bargaining power goes to zero, i.e. $\beta = 0$ so that workers appropriate nothing, nearly zero, of the surplus. It is important to understand, however, that multiple equilibrium do not occur for all possible parameter configurations.

3.3. Is heterogeneity the cause of multiplicity?

By comparing the equilibrium part in two types case and the first part of the paper, the immediate question is raised as to whether heterogeneity is the cause of multiplicity? To answer this question, suppose workers are homogenous, in order that multiple equilibria occurs, it must be check that $\Omega(.)$ is increasing at θ. That is:

$$\frac{\theta m'(\theta) - m(\theta)}{\theta^2} \frac{(x^*(a_h, \theta) - b)(1-\beta)}{\beta m(\theta) + r + \delta + \phi} + \frac{m(\theta)}{\theta}$$

$$\frac{\beta(r+\delta+\phi)}{r+\phi} \frac{m'(\theta)}{(\beta m(\theta) + r + \delta + \phi)^3} \frac{1-\beta}{C_{xx}} > 0$$

where C_{xx} is the second derivative of productivity cost function w.r.t productivity. As the expression for optimal productivity choice is:

$$\frac{\partial C(x^*)}{\partial x} = \frac{\beta m(\theta)}{(r+\phi)(\beta m(\theta) + r + \delta + \phi)} \qquad (3.15)$$

Inserting the above expression for $C_x\big(x^*(\theta; a)\big)$, gives

$$\frac{m(\theta) - \theta m'^{(\theta)}}{\theta} \frac{x^* - b}{m'(\theta)(r+\delta+\phi)} \frac{(\beta m(\theta) + r + \delta + \phi) C_{xx}}{C_x} \leq 1 \qquad (3.16)$$

By looking at above equation one can claim that if optimal productivity choice is close to unemployment benefit b then $\frac{\partial \Omega}{\partial \theta} > 0$. Note that x^* close to b contradict the active search constraint, so heterogeneity is not the cause of multiplicity.

4. Social Planner's Problem

Following Hosios (1990), I solve the social planner problem which determines the efficient allocation on the above economy. I assume the planner's discount rate equals to zero. By allowing this assumption I simplify the analysis to compare steady-state solutions rather than having to determine the discounted value of the change in some variable along the convergent path from one solution to another. The Planner chooses $\{E_i, U_i^A, V, x_i\}$ to maximise steady state aggregate net of output minus productivity investment and search cost in the economy, where E_i is the number of type i employed worker, U_i^A is the number of type i active unemployed workers , V is the number of total vacancies in the economy and x_i refers to the flow output produced by low/high ability worker.For simplicity the planner problem is:

$$\max_{E_i, x_i, U_i^A, V} \mathbb{P} = \sum_{\mathrm{i=l,h}} (E_i x_i + [\eta_i - E_i] b) - \sum_{\mathrm{i=l,h}} \left(\phi\left[U_i^A + E_i\right]\right) C(x_i; a_i) - kV. \tag{4.1}$$

Welfare is the sum of output produce by active low/high ability job seekers net of unemployment benefit to the inactive unemployed workers, minus the sum of investment productivity costs for both types and the total cost of posting vacancies in the economy. The planner should maximizes P subject to the steady sate turnover:

$$U_i^A\left(\phi + m(\theta)\right) = \delta E_i + \phi\left(U_i^A + E_i\right) \;\; for \;\; i = l, h \tag{4.2}$$

where $\theta = \frac{V}{U_l^A + U_h^A}$. Since $m(\theta)$ is the arrival rate of vacancies and ϕ is arrival rate of new entrants , the flow of low/high ability job seekers out of unemployment is $(m(\theta) + \phi) U_i^A$. The corresponding flow into active unemployment is $\delta E_i + \phi\left(U_i^A + E_i\right)$ where E_i is the number of low/high ability employed worker in the labor market. Lets define a control $\lambda_i = \frac{E_i + U_i^A}{\eta_i}$ where $0 \leq \lambda_i \leq 1$. λ_i defines the proportion of type i who are active. Clearly if both types choose $\lambda_i = 1$, then the number of unemployed workers is same as number of active workers in this economy. Consequently the proportion of active workers who are low/high ability type is equal to the number of workers of low/high type . Also if $\lambda_i = 0$ then no one participate in the labor market ,i.e. $E_i = -U_i^A = 0$. Using the expression for π_i where

$$\pi_i = \frac{\eta_i \left[\frac{E_i + U_i^A}{E_i + U_i}\right]}{\eta_l \left[\frac{U_l^A + E_l}{E_l + U_l}\right] + \eta_h \left[\frac{U_h^A + E_h}{U_h + E_h}\right]}$$

which denotes the proportion active agents who are low ability in the economy , if $\lambda_i = 0$ then $\pi_i = 0$ and for the case that $\lambda_i = 1$ then $\pi_i = \eta_i$. Given λ_i and the steady state turnover constraint (37) then the planner's problem equation (36) reduces to:[1]

$$\max_{x_i,\lambda_i,\theta} \mathbb{P} = \sum\nolimits_{\mathrm{i=l,h}} \frac{\lambda_i \eta_i m(\theta)}{\phi + \delta + m(\theta)} x_i + \sum\nolimits_{\mathrm{i=l,h}} \left[\eta_i - \frac{\lambda_i \eta_i m(\theta)}{\phi + \delta + m(\theta)}\right] b - k \frac{\theta(\phi + \delta)}{m(\theta) + \phi + \delta} \sum\nolimits_{\mathrm{i=l,h}} (\lambda_i \eta_i) - \sum\nolimits_{\mathrm{i=l,h}} \phi[\eta_i \lambda_i C(x_i; a_i)] \,. \tag{4.3}$$

This is the standard optimization problem solved by the Lagrangian method. The necessary conditions for optimality are described in Appendix 2.

In the standard matching model, the decentralized allocation is inefficient unless the so-called Hosios Condition holds.[2] To highlight the novel inefficiency, a series of possible optimal productivity investment, labor market participation and vacancy creation decision externalities are explained in the next section. Later I assume the planner has three tools to alter the market outcome.

4.1. Efficiency

4.1.1. Socially efficient labor market tightness

Using the first order conditions presented in the previous part, I solve the efficient labor market tightness θ.

Proposition 3: The socially efficient labor market tightness is given by:

$$k = \frac{m'(\theta)}{(m(\theta) + \phi + \delta - m'(\theta)\theta)} \left[\frac{\lambda_l \eta_l}{\lambda_l \eta_l + \lambda_h \eta_h} x_l + \frac{\lambda_h \eta_h}{\lambda_l \eta_l + \lambda_h \eta_h} x_h - b\right] \tag{4.4}$$

1. The detailed solution of Planner problem is in the Appendix.
2. This Condition states that without a capital choice, the equilibrium is optimal if and only if the worker's bargaining share is equal to the elasticity of the matching function with respect to the number of vacancies.

The Hosios rule sets the worker share of the net surplus equal to the elasticity of the matching function with respect to unemployment. It can be written

$$1-\beta = \frac{m'(\theta)\theta}{m(\theta)}. \tag{4.5}$$

Previously the paper showed that the labor market tightness in the decentralized case without policy is given by:

$$k = \frac{m(\theta)(1-\beta)}{\theta\big(r+\phi+\delta+\beta m(\theta)\big)}\big[\pi(\theta)x^*(a_l,\theta) + \big(1-\pi(\theta)\big)x^*(a_h,\theta) - b\big] \tag{4.6}$$

If the Hosios condition holds then, given that participation is efficient then the decentralized free entry condition will be equal to the planner solution (see Appendix 2).

Consider those individuals that are indifferent to participate in labor market, from the participation constraint lets substitute $x_i - b$ into above market and planner free entry conditions then:

$$k = \frac{m'^{(\theta)\phi}}{m(\theta)-m'(\theta)\theta}[\pi C(x_l;a_l) + (1-)\, C(x_h;a_h)] \quad Planner\ solution,$$

$$k = \frac{(1-\beta)\phi}{\beta\theta}[\pi C(x_l;a_l) + (1-\pi)C(x_h;a_h)] Market\ solution.$$

Equating the corresponding social planner and market productivity's investment decision gives $\frac{m'(\theta)\theta}{m(\theta)} = 1-\beta$. Observe that if the worker is indifferent to participate in the labor market then Hosios Condition will be satisfied. This result extends Hosios'(1990) results, which showed that without a capital choice, the equilibrium is optimal if and only if the worker's bargaining share is equal to the elasticity of the matching function, however, with endogenous capital investment, this bargaining share leads to hold up problems, as shown previously. At the root of excessive of posting vacancies result is the fact that firms create a negative externality when they enter, since they make it harder for the other firms to find workers. Simultaneously, they create positive externality on workers irrespective of their abilities because they increase the probability that workers find employment. Basically increase entry of low ability workers’ participation imposes a diseconomy on existing participants and external economy on firms. Hence, firms create more vacancies, leading to further vacancy creation and so on. The balance of these forces is ambiguous in general but depends on the relative share of surplus going to workers and firms and the optimal productivity investment of the workers according to their ability and cost of investment.

4.1.2. Socially efficient productivity investment

Using the first order conditions of the planner problem, I can solve the socially efficient productivity levels x_i.

Proposition 4: The socially efficient productivity investment x_i is given by:

$$\frac{\partial C}{\partial x_i} = \frac{m(\theta)}{\phi\big(\phi + \delta + m(\theta)\big)} \tag{4.7}$$

Proof. Solving the necessary condition for optimality complete the proof.

I have shown that the solutions for the decentralised case is given by:

$$\frac{\partial C}{\partial x}(x_i^*(\theta), a_i) = \frac{m(\theta)\beta}{\big(r + \phi + \delta + \beta m(\theta)\big)(r + \phi)} \tag{4.8}$$

Observe[1] that (42) represents the marginal cost of productivity investment when the planner choose x optimally. Whereas (43) represents marginal cost of productivity investment when the individual chooses her/his productivity optimally. The difference between these solution is in parameter β which is the worker bargaining power. Investment in productivity reveals the hold up problem. Hold up arise because workers must invest in productivity before meeting a firm, and firms may reap some of the benefits from larger investments. Therefore, the corresponding social and private marginal investment solutions are equal if and only if worker has got full bargaining power. When individuals make ex ante investments before matching with firms disregard their ability and also wages are determined by ex post bargaining, the equilibrium is inefficient.[2] Wages increase with productivity investment, creating hold up problem for unemployed active job seekers' investment, also all the bargaining power is controlled by the workers leading

1. As agents optimal expenditure decisions ignore the share to be obtained by their trading partners, agents's search and recruitment expenditures are inefficient, Mortensen (1982a).
2. This result is related to Acemoglu and Shimer (1999b) findings.

to very high wage level and excessive entry of workers. Clearly with ex ante investments , no bargaining solution achieves efficiency . It is often emphasized that human capital externalities raise output at the aggregate level, here it is clear that the social solution exceed the market solution unless the worker's bargaining power is equal to one.

4.1.3 Socially efficient participation level

Using the first order conditions presented in Appendix 1, I can solve the socially efficient participation level λ_i.

Proposition 5: From the social point of view, an individual with productivity x_i will participate in the labor market only if $x_i > b + \frac{\phi\left(\phi+\delta+m(\theta)\right)}{m(\theta)} C(x_i) + \frac{k\theta(\phi+\delta)}{m(\theta)}$.

The standard search model features a fixed size of the labor force [see, for instance , Pissarides (2000)] while here I endogeny the labor force participation. With fixed participation, Hosios (1990) showed that the wage rule decentralizes the efficient labor market allocation if and only if the bargaining power of the worker equals the elasticity of the number of aggregate matches with respect to the number of individuals searching for the jobs. While the supplies of labor have been endogenous, we are able to determine whether their incentives for entry are efficient. In this case, we can determine the parameter λ_i which shows the proportion of high/low ability workers who are active that is those who participate in the labor market . Diamond (1982b) argue that the presence of an additional worker(firm) makes the entry easier(harder) for vacancies to find workers but harder(easier) for workers to find jobs. Observe that labor heterogeneity and cost of investing on productivity make the additional source of inefficiency from those identified by the matching literature. As I showed before the participation decision of market solution is given by:

If $x_i > b + \frac{\phi(\phi+\delta+\beta m(\theta))}{\beta m(\theta)} C(x_i^*)$ then participate.

In simple words , given the efficient optimal productivity investment, efficient labor market participation requires no cost of posting vacancy which can be concluded from equating the numerator of the last part planner participation that is $\frac{k\theta(\phi+\delta)}{m(\theta)}$ to zero.

In the traditional search and matching models we have two traditional externalities. When firms enter the market, they make it harder for other firms to find workers, so a negative externality happens (congestion externality), but since they increase the probability that workers find employment a positive externality on workers happens (thick market externality)[see Pissarides (2000)] . These two externalities cancel each other under the Hosios condition . Notice that in my model I find additional externality called "composition externality". It is created by the different types of workers searching for a job with different productivity investment. Therefore, labor market is overcrowded with low productivity workers who search for a job and reduce the probability that high productivity workers match. Clearly these externalities makes the decentralized solution inefficient.

4.2. Policy implications

The next step is to examine whether policy can improve on the decentralized allocation. In this case the government can uses different policy instruments. I study three policy instruments that may allow us to achieve the *First Best* in the economy. I assume that the government apply the principle of targeting[1] and implement the following policies: training subsidy s for those who invest on their education, labor market participation tax t and finally a job creation subsidy z. It is also of interest to know more generally how, these policies interact with worker bargaining power to affect efficiency.

4.2.1. Optimal training subsidy policy

This paper first introduces the optimal training subsidy and show that how it interact with worker bargaining power to affect efficiency. Let us introduce e_i which is the difference between the inherent ability a_i and the ex-post productivity x_i of the worker, i.e. $x_i = a_i + e_i$. The productivity investment cost $C(x_i; a_i)$ Features the same as

$$C(a_i + e_i; a_i) \equiv \hat{C}(e_i; a_i)$$

So, we can simply define

1. The generalization of the principle of targeting is in line with Dixit, Grossman and Helpman (1996). In their common agency model, more efficient instruments are chosen because the government cares about social welfare.

$$C(x_i; a_i) = C(a_i + e_i; a_i) \equiv \widehat{C}(e_i; a_i).$$

Introducing the optimal training subsidy per unit of investment s applied by the government leads to:

$$[1 - s]\frac{\partial \hat{C}}{\partial e_i}(e_i; a_i) = \frac{\beta m(\theta)}{\phi\big(\phi + \delta + \beta m(\theta)\big)} \tag{4.9}$$

Proposition 6: The optimal training subsidy that targets the efficient productivity investment decision level is given by:

$$s^* = 1 - \frac{\beta\big(\phi + \delta + m(\theta)\big)}{(\phi + \delta + \beta m(\theta))} \tag{4.10}$$

Proof. Substituting (4.7) into (4.9) and rearranging the terms, I find the optimal policy in proposition (5).

COROLLARY. The optimal education subsidy rate is given:

If $\beta = 1$ then $s^* = 0$ and

If $\beta < 1$ then $s^* = \frac{(1-\beta)(\phi+\delta)}{\phi+\delta+\beta m(\theta)} > 0$.

Assuming $\beta = 1$ then the optimal education subsidy will be equal to zero and it eliminates the investment decision externality. For the case that worker has some bargaining power but not the full , then optimal training policy is positive . It turns out that a higher worker's bargaining power leads to implement a lower training subsidy in order to restore investment decision efficiency. One way of achieving productivity investment decision efficiency is to raise worker bargaining power so that worker appropriate all of the surplus. This formalizes the notation that efficiency requires a solution to the holdup problem. Since firms do not share in the cost of ex-ante productivity investments, this leads to underinvestment.

4.2.2. Participation tax policy

I now turn to a formal analysis of the effects of participation fee(tax) entry policy. The reason of introducing this policy is to deter the individuals with low ability to participate in the labor market. Let t be the lump-sum fee entry regardless of skill, so an active unemployed worker value function with this policy is given by:

$$(r+\phi)V_U(x,\theta) = b - t + m(\theta)[V_E(x,\theta) - V_U(x,\theta)] \tag{4.11}$$

and when employed,

$$(r+\phi)V_E(x,\theta) = w(x,\theta) + \delta[V_U(x,\theta) - V_E(x,\theta)]. \tag{4.12}$$

It is simple to show that imposing the participation fee policy the wage will be

$$w^p = \frac{x(r\phi + \delta + m(\theta)\beta) + (b-t)(1-\beta)(r+\phi+\delta)}{(r+\phi+\delta+\beta m(\theta))} \tag{4.13}$$

and accordingly, the value of being active unemployed worker substituting (4.13) into (4.11) gives:

$$V_U(x,\theta) = \frac{(b-t)(r+\phi+\delta) + m(\theta)\beta x}{\big(r+\phi+\delta+\beta m(\theta)\big)(r+\phi)}. \tag{4.14}$$

Proposition 7: Using the optimal productivity investment policy s^*, the optimal labor market tax participation is given by the following condition:

$$t^* = \beta k\theta + \frac{\phi}{\phi+\delta}\big[(\phi+\delta)(\beta-1) + s\big(\phi+\delta+\beta m(\theta)\big)C(x;a_i)\big] \tag{4.15}$$

<u>Proof.</u> Using the decentralized participation constraint and substitution gives,

$$x > b + \frac{t(r+\phi+\delta)}{m(\theta)\beta} + \frac{\big(r+\phi+\delta+\beta m(\theta)\big)(r+\phi)(1-s^*)}{m(\theta)\beta} \tag{4.16}$$

given the optimal training subsidy policy solution, comparing with the social planner solution, i.e.

$$x > b + \frac{k\theta(\phi+\delta)}{m(\theta)} + \frac{\phi(\phi+\delta+m(\theta)}{m(\theta)}C(x;a_i),$$

rearranging the terms, I find the solution in proposition (5).

If $\beta = 1 \Rightarrow s^* = 0 \Rightarrow t^* = k\theta$, the first thing to note is that if $\beta = 1$ (i.e. worker bargaining power is full) $s^* = 0$ as claimed, so that the optimal labor market participation fee will be equal to $k\theta$. It is also apparent that if β is less than one, the optimal fee policy will be $\beta k\theta$. The term $k\theta$ is commonly interpreted as the value of saved hiring costs due to the existence of an additional matched worker, and for the case that β is less than one we can see the optimal fee entry is $\beta k\theta$ which often interpreted as the capitalised value built into negotiated wage , shared according the worker's bargaining power. One might conjecture from this that introducing a lump-sum tax entry will be ineffective while it would discourage workers to participate in labor market. It is interesting to know how this type of taxation interact with worker bargaining power in equilibrium to affect efficiency. It turns out that these two policies are closely related.[1]

4.2.3. Job creation subsidy policy

Now suppose that some of the fee entry is redistributed as lump-sum payments to firms to subsidize the cost of posting a vacancy. Let this subsidy be z . The value of a vacant job is:,

$$rJ_V(\theta) = -k + z + \frac{m(\theta)}{\theta}[\pi(J_F(x_l^*(\theta) - J_V) + (1 - \pi)(J_F(x_h^*(\theta)) - J_V(\theta)]$$

Proposition 8: Using the optimal policies s^* and t^*, the optimal job creation subsidy z^* is given by:,

$$z^* = \{\left[\frac{m'(\theta)}{(m(\theta) + \phi + \delta - m'(\theta)\theta)} - \frac{m(\theta)(1-\beta)}{\theta\big(r + \phi + \delta + \beta m(\theta)\big)}\right] \times \left[\frac{\lambda_l\eta_l}{\lambda_l\eta_l+\lambda_h\eta_h}(x_l - b) + \frac{\lambda_h\eta_h}{\lambda_l\eta_l+\lambda_h\eta_h}(x_h - b)\right] - \beta k m(\theta)1 - \frac{\beta}{\phi} + \delta + \beta m(\theta).\} \tag{4.17}$$

Of course this is the solution of the case that β is strictly between 0 and 1. Clearly, z^* is positive as long as the multiplication of the two brackets on the right hand side of equation (52) is greater than $\frac{\beta k m(\theta)(1-\beta)}{\beta m(\theta)+\phi+\delta}$.

1. Bovenberg and Jacobs (2005) consider optimal tax policy where the government taxes labor income but, as workers also underinvest in education, government offers education subsidies.

Finally, when worker has full bargaining power the optimal job creation subsidy is equal to the social planner's cost of creating a vacancy. That is,

$$z^* = \left(\frac{m'(\theta)}{m(\theta) + \phi + \delta - m'(\theta)\theta}\right)\left[\frac{\lambda_l \eta_l}{\lambda_l \eta_l + \lambda_h \eta_h}(x_l - b) + \frac{\lambda_h \eta_h}{\lambda_l \eta_l + \lambda_h \eta_h}(x_h - b)\right].$$

One might conjecture how introducing participation fee τ, job creation subsidy z and training subsidy s in equilibrium interact with worker bargaining power to affect efficiency. It turns out that these optimal policies are closely related. Applying the principle of targeting shows when workers achieve all the surplus the optimal training subsidy is equal to zero and the optimal participation tax will be hence $k\theta$. Therefore the optimal job creation subsidy is positive. If the Hosios condition holds, then given that participation is efficient then the decentralized, free entry condition will be equal to the planner solution.

5. Conclusions

Workers and firms face considerable problem contacting each other and of course these difficulties have consequences on the equilibrium characteristics of the labor market. This paper studies an equilibrium search model that highlights the role of inherent ability and productivity investment in the labor market . The argument is related to the endogenous participation and investment decision of heterogeneous workers who have an inherent ability level. When productivity investment is costly and workers are heterogeneous in ability one can think only the ablest choose to acquire productivity (education). While here I show the important role of critical ability and cost of investing on productivity which makes the result different.

Clearly the active constraint plays an important role in the analysis and makes it possible to examine the interaction between critical ability and market tightness and also the choice of optimal productivity investment. I show here those with ability above critical level will participate in the labor market. The choice of participation involves an opportunity costs in terms of forgone utility of leisure and direct cost of productivity investment . I prove that the critical ability decreases in market tightness while optimal productivity investment increases in market tightness and ability of the worker.

Embedding the wage bargaining with free entry condition, optimal productivity investment and participation decision, I describe equilibrium characteristics. I show the existence of equilibrium and then extend the model for two types of workers since it gives a better description of the equilibrium and probability of existing multiple equilibrium. Equilibrium can take one of the following forms. One in which it is beneficial for both types of workers to be active in the labor market, I call it the “Joint Type Equilibrium”, second one is the one that only there is willingness of high ability workers to invest in their productivity and finally the last one is the one where there is no benefit for either types to invest in their productivity and consequently there is no participation of workers in the labor market and no posting of vacancies from the firm side.

The paper shows that the market solution is not efficient, since workers and firms do not internalize the cost of posting a vacancy of the firms, participation decision and productivity investment of the workers. The market solution implies that the productivity investment of the worker is lower than the planner's solution which reveals the holdup problem. It arises because worker must invest in productivity before meeting a firm and firm reaps some of the benefits from the worker's investment. The decentralized solution implies that workers with low productivity will participate in the labor market, therefore job creation and labor market tightness will not be equal to social planner case. Therefore, the number of workers with low productivity in the economy is high and the job creation is low.

Since the market solution is not efficient, optimal policies are required. Assuming the government observes the worker's eduction, I consider participation labor market tax policy where the government taxes participant workers, as workers underinvest in education, it in addition offers education subsidies. I show that training subsidy for those who decide to participate in the labor market increases the incentive to invest on productivity. On the other hand, training subsidy for those who are low ability type will increase incentive to be active. The introduction of a participation tax will have a perverse effects; it deters workers incentive to enter to the labor market since it is a kind of tax(fee) required to be paid as soon as he/she enters labor market.

This effect is more obvious for workers with low ability, it reduces the incentive for them to participate in the labor market. Of course the income flow of inactive unemployed workers(unemployment benefit) is an important determinant of the participation and investment decision. Not surprisingly taxes on labor market participation and training subsidy distort human capital investment and participation decision at different ability levels. But these distortions are potentially high for those individuals at the participation margin, whose abilities are close to threshold (critical) level, i.e. who are indifferent to participate in the labour market. The other additional insight from the paper is that the planner, using principle of targeting internalizes the externality by means of the efficient instrument, i.e. the one that aims directly at the source.

From the arguments of proceeding paper it is clear for the two types case of workers that the degree of inefficiency of equilibrium turns on the degree of worker bargaining power, β. I remark throughout the paper that one way of achieving efficiency is to raise worker bargaining power so that workers appropriate all of the surplus.

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Appendices

Appendix 1: Social Planner Problem

Lets assume $r = 0$, Planner maximises steady state flow payoffs. People die at rate $\phi \ \Rightarrow \phi$ is entry rate.

Lets define λ_i the proportion of type $i \in l, h$ who are active; i.e. , $\frac{U_i^A + E_i}{\eta_i}$. Clearly if both types choose $\lambda_i = 1$ then the number of unemployed workers is the same as the number of active unemployed workers in this economy. Consequently the proportion of active workers who are low/high type is equal to the number of workers of low/high type. Also if $\lambda_i = 0$ then $\pi_i = 0$.[1]

The objective function of the social planner is:

$$\max_{\eta_l, \eta_h, \lambda_l, \lambda_h, x_l, x_h, U_l, U_h, V} \text{P} = (\eta_l - U_l)x_l + U_l b + (\eta_h - U_h)x_h + U_h b - \phi[\eta_l \lambda_l C(x_l; a_l) + \eta_h \lambda_h C(x_h; a_h)] - kV \tag{1}$$

subject to three steady state turnovers by the following equations:

$$(m(\theta) + \phi)U_i^A = \delta E_i + \phi \eta_i \lambda_i \tag{2}$$

$$\phi\left(U_i - U_i^A\right) = \phi \eta_i (1 - \lambda_i) \tag{3}$$

$$U_i + E_i = \eta_i \tag{4}$$

and also the market tightness condition:

$$\theta = \frac{V}{U_l^A + U_h^A} \tag{5}$$

1. $\pi_i = \dfrac{\eta_i \left[\frac{E_i + U_i^A}{E_i + U_i}\right]}{\eta_l \left[\frac{U_l^A + E_l}{E_l + U_l}\right] + \eta_h \left[\frac{U_h^A + E_h}{U_h + E_h}\right]}$.

The first steady state condition; (2) is that the flow of low/high ability active unemployed worker out of unemployment equals the flow of low/high ability unemployed worker back into active unemployment. Since $m(\theta)$ is the arrival rate of vacancies and ϕ is arrival rate of new entrants, the flow of high/low ability unemployed active workers out of unemployment is $(m(\theta)+\phi)U_i^A$. The corresponding flow into active unemployment is $\delta E_i + \phi\eta_i\lambda_i$. The second steady state condition is that the flow of inactive workers out of unemployment equals the flow of inactive workers into unemployment. There are η high/low ability workers, of whom $1-\lambda$ are inactive. So the flow into inactive low/high ability unemployment would be $\phi\eta_i(1-\lambda_i)$. Moreover total number of workers having high/low ability is η which is sum of low/high employed and unemployed workers in third steady sate condition. Substituting (3) and (4) into (2) gives,

$$U_i^A\big(\phi + m(\theta)\big) = \delta E_i + \phi\big(U_i^A + E_i\big) \quad for\ \ i = l, h \tag{6}$$

which implies that

$$U_i^A = \frac{(\phi+\delta)E_i}{m(\theta)} \tag{7}$$

Lets define

$$\lambda_i = \frac{E_i + U_i^A}{\eta_i} \tag{8}$$

Substituting (7) into (8) and rearranging in terms of E_i gives,

$$E_i = \lambda_i\eta_i \frac{m(\theta)}{\phi+\delta+m(\theta)} \tag{9}$$

Substituting (9) into (1) the planner problem reduces to

$$\max_{x_i,\lambda_i,\theta} \mathbb{P} = \frac{\lambda_l\eta_l m(\theta)}{\phi+\delta+m(\theta)} x_l + \frac{\lambda_h\eta_h m(\theta)}{\phi+\delta+m(\theta)} x_h + \left[\eta_l - \frac{\lambda_l\eta_l m(\theta)}{\phi+\delta+m(\theta)}\right] b + \tag{10}$$

$$\left[\eta_h - \frac{\lambda_h \eta_h m(\theta)}{\phi + \delta + m(\theta)}\right] b - k \frac{\theta(\phi + \delta)}{m(\theta) + \phi + \delta} (\lambda_l \eta_l) + (\lambda_h \eta_h) - \phi[\eta_l \lambda_l C(x_l; a_l)] - \phi[\eta_h \lambda_h C(x_h; a_h)]$$

I can rewrite (11) function as:

$$\max_{x_i, \lambda_i, \theta} \mathbb{P} = \sum_{\text{i=l,h}} \frac{\lambda_i \eta_i m(\theta)}{\phi + \delta + m(\theta)} x_i + \sum_{\text{i=l,h}} \left[\eta_i - \frac{\lambda_i \eta_i m(\theta)}{\phi + \delta + m(\theta)}\right] b - \tag{11}$$

$$k \frac{\theta(\phi + \delta)}{m(\theta) + \phi + \delta} \sum_{\text{i=l,h}} (\lambda_i \eta_i) - \sum_{\text{i=l,h}} \phi[\eta_i \lambda_i C(x_i; a_i)]$$

Appendix 2: Optimal Policies

Using the Lagrangian Method I can solve this standard optimization problem as: This problem satisfies the following first order conditions:

$$\frac{\partial \mathbb{P}}{\partial x_i} = \frac{\partial C}{\partial x_i} - \frac{m(\theta)}{\phi\big(\phi + \delta + m(\theta)\big)} = 0 \tag{1}$$

$$\frac{\partial \mathbb{P}}{\partial \lambda_i} = x_i - b - \frac{\phi(\phi + \delta + m(\theta))}{m(\theta)} C(x_i; a_i) - \frac{k\theta(\phi + \delta)}{m(\theta)} = 0 \tag{2}$$

$$\frac{\partial \mathbb{P}}{\partial \theta} = k - \frac{m'^{(\theta)}}{(m(\theta) + \phi + \delta - m'(\theta)\theta)} \left[\frac{\lambda_l \eta_l}{\lambda_l \eta_l + \lambda_h \eta_h} x_l + \frac{\lambda_h \eta_h}{\lambda_l \eta_l + \lambda_h \eta_h} x_h - b\right] = 0 \tag{3}$$

Rearranging (12)

$$\phi \eta_i \lambda_i = (m(\theta) + \phi) U_i^A - \delta\big(\eta_i \lambda_i - U_i^A\big) \tag{4}$$

$$\eta_i \lambda_i = \left(\frac{m(\theta)}{\phi + \delta} + 1\right) U_i^A \tag{5}$$

Consider those individuals that are indifferent to participate in labor market, from the participation constraint lets substitute $x_i - b$ into above market and planner free entry conditions then:

$$k = \frac{m'^{(\theta)\phi}}{m(\theta) - m'(\theta)\theta}[\pi C(x_l; a_l) + (1-\pi)C(x_h; a_h)] \; Planner\ solution \tag{6}$$

$$k = \frac{(1-\beta)\phi}{\beta\theta}[\pi C(x_l; a_l) + (1-\pi)C(x_h; a_h)] Market\ solution \tag{7}$$

when the worker is indifferent to participate equality holds then

$$\frac{m'(\theta)\phi}{m(\theta) - m'(\theta)\theta} = \frac{(1-\beta)\phi}{\beta\theta}$$

then

$$\frac{m'(\theta)}{m(\theta) - m'(\theta)\theta} = \frac{1-\beta}{\beta\theta}$$

dividing both sides by $m(\theta)$

$$\frac{\frac{m'}{m(\theta)}}{1 - \frac{m'\theta}{m(\theta)}} = \frac{1-\beta}{\beta\frac{m(\theta)\theta}{m'(\theta)}}$$

Rearranging the terms proves that Hosios Condition, i.e.; $1 - \beta = \frac{m'(\theta)\theta}{m(\theta)}$ satisfies.